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ADIABATICS OF SUPERHEATED STEAM

BY

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THESIS FOR THE DEGREE OF BACHELOR OF SCIENCE

IN MECHANICAL ENGINEERING

IN THE

COLLEGE OF ENGINEERING

OF THE

UNIVERSITY OF ILLINOIS

JUNE, 1910

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THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

William Gottlieb Hiller

ENTITLED Adiabatics of Superheated Steam

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Bachelor of Science in Mechanical Engineering

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ADIABATICS OF SUPERHEATED STEAM

I INTRODUCTION

The rapid development in the use of superheated steam within recent years has made it highly desirable that there should be available an easy means for making calculations which arise in engine and turbine practice. All of the formulae now available for making these calculations are exceedingly difficult to apply in practice, and further, the conflicting nature of the of-the experimental data upon which they are based cast doubt upon their reliability. It would seem that tables of the properties of superheated steam such as those by Professor Peabody and by Professor Marks and Dr. Davis would simplify such calculations, but these again are subject to the objection that they were obtained by the graphical integration of assumed specific heat curves.

In a recent paper by Professor Goodenough a connected theory has been evolved, by means of which the important properties of superheated steam can be calculated directly without recourse to approximation. This theory is based upon the most reliable experimental data available.

(a) EQUATIONS FOR SUPERHEATED STEAM.

The following derivation of equations is taken from the above mentioned paper by Professor Goodenough.

"From the general principles of thermodynamics we have the relation

$$\left[\frac{\delta c_p}{\delta p} \right]_T = -AT \frac{\delta^2 v}{\delta T^2} \quad \text{-----} \quad (1)$$

which applies to all substances. Writing the characteristic equation

$$pv = BT - (1 + ap) \frac{m}{T^n} p, \quad \text{-----} \quad (2)$$

in the form

$$v = \frac{BT}{p} - (1 + ap) \frac{m}{T^n}, \quad \text{-----} \quad (3)$$

we readily obtain by differentiation,

$$\frac{\delta v}{\delta T} = \frac{B}{p} + (1 + ap) \frac{mn}{T^{n+1}},$$

$$\frac{\delta^2 v}{\delta T^2} = -(1 + ap) \frac{mn(n+1)}{T^{n+2}},$$

whence for superheated steam (1) becomes

$$\left[\frac{\delta c_p}{\delta p} \right]_T = (1 + ap) \frac{Amn(n+1)}{T^{n+1}} \quad \text{-----} \quad (4)$$

Taking T as a constant and integrating (4) with p as the independent variable the result is

$$c_p = (1 + \frac{ap}{2}) p \frac{Amn(n+1)}{T^{n+1}} + \text{constant of integration.}$$

The constant of integration may be a function of T , since T was held constant. Hence

$$c_p = \phi(T) + (1 + \frac{ap}{2}) \frac{Amn(n+1)}{T^{n+1}} p \quad \text{-----} \quad (5)$$

or

$$c_p = \phi(T) + f(p, T), \quad \text{--- (6)}$$

if the term

$$\left(1 + \frac{a}{2} p\right) \frac{A m n (n + 1)}{T^{n+1}} p$$

is replaced for convenience by $f(p, T)$

From a consideration of the most reliable experimental data available, Professor Goodenough shows that the function $\phi(T)$ has the form $\alpha + \beta T$ for temperatures usually employed in engine and turbine practice. Whence, substituting in (5)

$$c_p = \alpha + \beta T + \left(1 + \frac{a}{2} p\right) \frac{A m n (n + 1)}{T^{n+1}} p \quad \text{--- (7)}$$

"With the aid of an explicit equation for the specific heat, expressions for the heat content, energy and entropy are readily derived. For this purpose the general equation

$$dq = c_p dT - A T \frac{\delta v}{\delta T}_p dp \quad \text{--- (8)}$$

is most convenient. Since heat content is defined by the relation

$$i = A (u + p v)$$

we have

$$di = dq + A v dp,$$

whence from (8)

$$di = c_p dT - A \left(T \frac{\delta v}{\delta T} - v \right) dp \quad \text{--- (9)}$$

From the characteristic equation (3) the derivative $\frac{\delta v}{\delta T}$ is found to be

$$\frac{\delta v}{\delta T} = \frac{B}{p} + (1 + ap) \frac{mn}{T^n + 1}.$$

Substituting this and also the expression for c_p in (9), the result is

$$di = (\alpha + \beta T) dT + Amn (n + 1) p (1 + \frac{a}{2} p) \frac{dT}{T^n + 1} + \frac{Amn (n + 1)}{T^n} (1 + ap) dp$$

The integral of this exact differential is

$$i = T + \frac{1}{2} T^2 - \frac{Amn (n + 1)}{T^n} p (1 + \frac{a}{2} p) + i_0 \quad \text{--- (10)}$$

"Since $Au = i - Apv$, we obtain for the energy expressed in thermal units

$$Au = T(\alpha + \frac{\beta}{2} T^2 - AB) - \frac{Amp}{T^n} \left[n + (n + 1) \frac{a}{2} p \right] + i_0 \quad \text{--- (11)}$$

"For the entropy s an expression is readily found from (8). Thus

$$ds = \frac{dq}{T} = c_p \frac{dT}{T} - A \frac{\delta v}{\delta T} dp \quad \text{--- (12)}$$

Introducing in this equation the expression previously derived for c_p and $\frac{\delta v}{\delta T}$, the result is

$$ds = \left(\frac{\alpha}{T} + \beta \right) dT + \frac{A m n (n + 1)}{T^{n+1}} p \left(1 + \frac{a}{2} p \right) \frac{dT}{T^{n+2}} - AB \frac{dp}{p} - \frac{A m n}{T^{n+1}} (1 + a p) dp \quad (13)$$

The integral is

$$s = \alpha \log_e T + \beta T - AB \log_e p - \frac{A m n p}{T^{n+1}} \left(1 + \frac{a}{2} p \right) + s_0 \quad (14)$$

The constants in the above equations have the following values as determined by Professor Goodenough.

English units.

$$\alpha = 0.372$$

$$\beta = 0.0001$$

$$\log m = 9.50308$$

$$n = 3.5$$

$$A = \frac{1}{777.7} = \frac{1}{J}$$

$$T = t + 459.6$$

$$B = 85.77 \quad (p \text{ in pounds per square foot})$$

$$= 0.5956 \quad (p \text{ in pounds per square inch})$$

$$i_0 = 882.4$$

$$s_0 = -0.4300$$

$$a = 0.0007$$

Reducing to common logarithms, and substituting the above constants, equation (14) reduces to the form

$$s = 0.85657 \log T + 0.0001 T - 0.25392 \log p - \frac{mnAp}{T^{n+1}} \left(1 + \frac{a}{2} p \right) - 0.4300 \quad \text{--- (15)}$$

(b) ADIABATIC EXPANSION OF SUPERHEATED STEAM.

A problem frequently met with in steam engineering is that of calculating the work done during the adiabatic expansion of steam, since the adiabatic expansion represents most nearly the actual change taking place in the cylinder of an engine or the stages of a turbine. It is highly desirable, therefore, that a simple method of making these calculations should be provided.

By the use of equation (14) we are enabled to find the pressure corresponding to any temperature during an adiabatic expansion, since s is then constant, and by substituting corresponding values of p and T in the modified form of the characteristic equation, (3), the corresponding value of v , the volume may be determined. However this is an exceedingly laborious process, involving as it does the solution of the transcendental equation (14). It is evident that if an exponential relation between pressure and volume of superheated steam could be found similar to that for a perfect gas, namely $pv^n = C$, and to that deduced by Zeuner for satur-

ated steam, the labor of calculating volumes, and work done during expansion would be materially decreased.

It is the purpose of the writer to derive if possible an exponent that will give results with an accuracy consistent with that of the data generally available in engineering problems.

II METHOD OF DETERMINING THE EXPONENT.

Equation (15) will be taken as the basis of this investigation. Selecting some convenient value of s on the T-s chart, (for this purpose the chart accompanying the Marks and Davis Steam Tables is convenient) the chosen value of entropy is substituted for s in equation (15), which then reduces to a transcendental with the two variables p and T . If either p or T is given an assumed value the other may be calculated. It seems most convenient for our present purpose to assume values of T and calculate the corresponding value of p . For a given value of s and T , equation (15) may be written

$$\log p = C - K \left(p + \frac{a}{2} p^2 \right) \text{ - - - - - (16)}$$

where $(0.85657 \log T + 0.0001 T - s - 0.4300) / 0.25392 = C$,

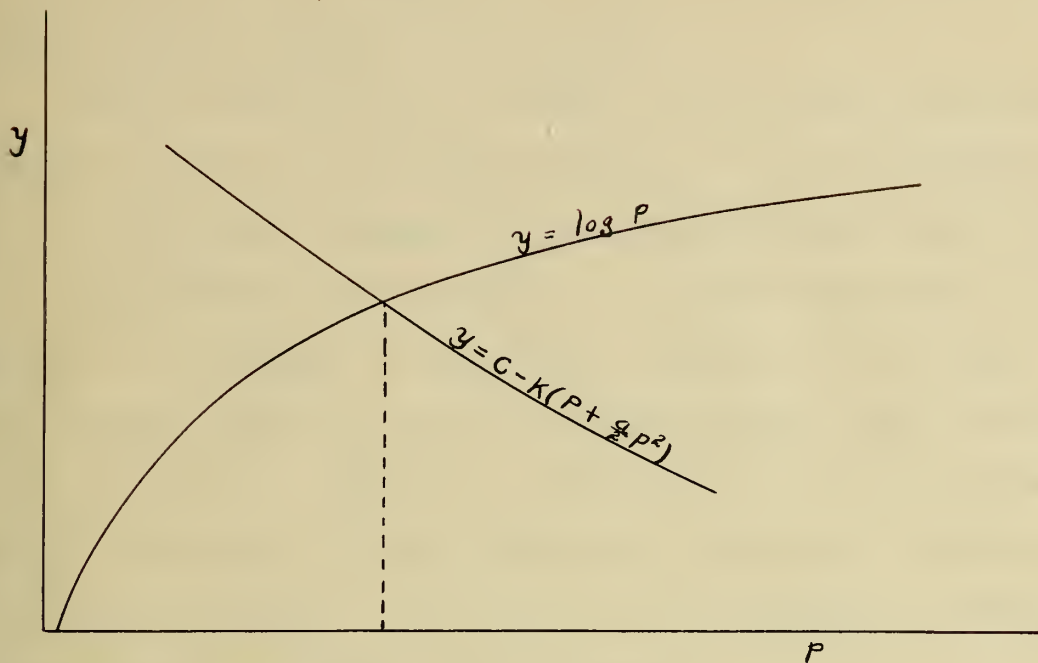
and

$$\frac{m n A}{0.25392 T^n + 1} = K.$$

The solution of equation (16) is accomplished most conveniently by writing it in the form

$$\log p = y = C - K \left(p + \frac{a}{2} p^2 \right),$$

and plotting the two curves as shown below.



Evidently the intersection of these curves gives the value of p that satisfies equation (16). As a check upon the solution, the value of p thus obtained is substituted back in equation (15). In a similar manner other values of p are obtained on the same adiabetic by using different values of T . In these calculations the value of T varies from 750° to 1500° by increments of 100° . The lower points being taken near the saturation line, and the upper points are higher than temperatures employed in practice.

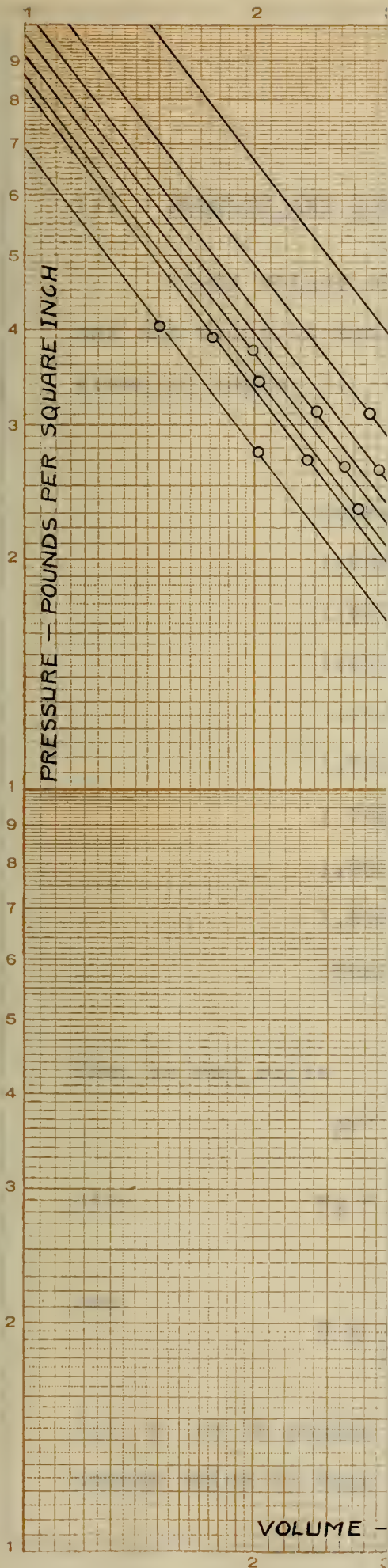
Having values of p and T on an adiabetic, the corresponding values of v may be found by substituting p and T in equation (3). Having a series of pressures and volumes on an adiabetic it becomes possible to determine the relation between them.

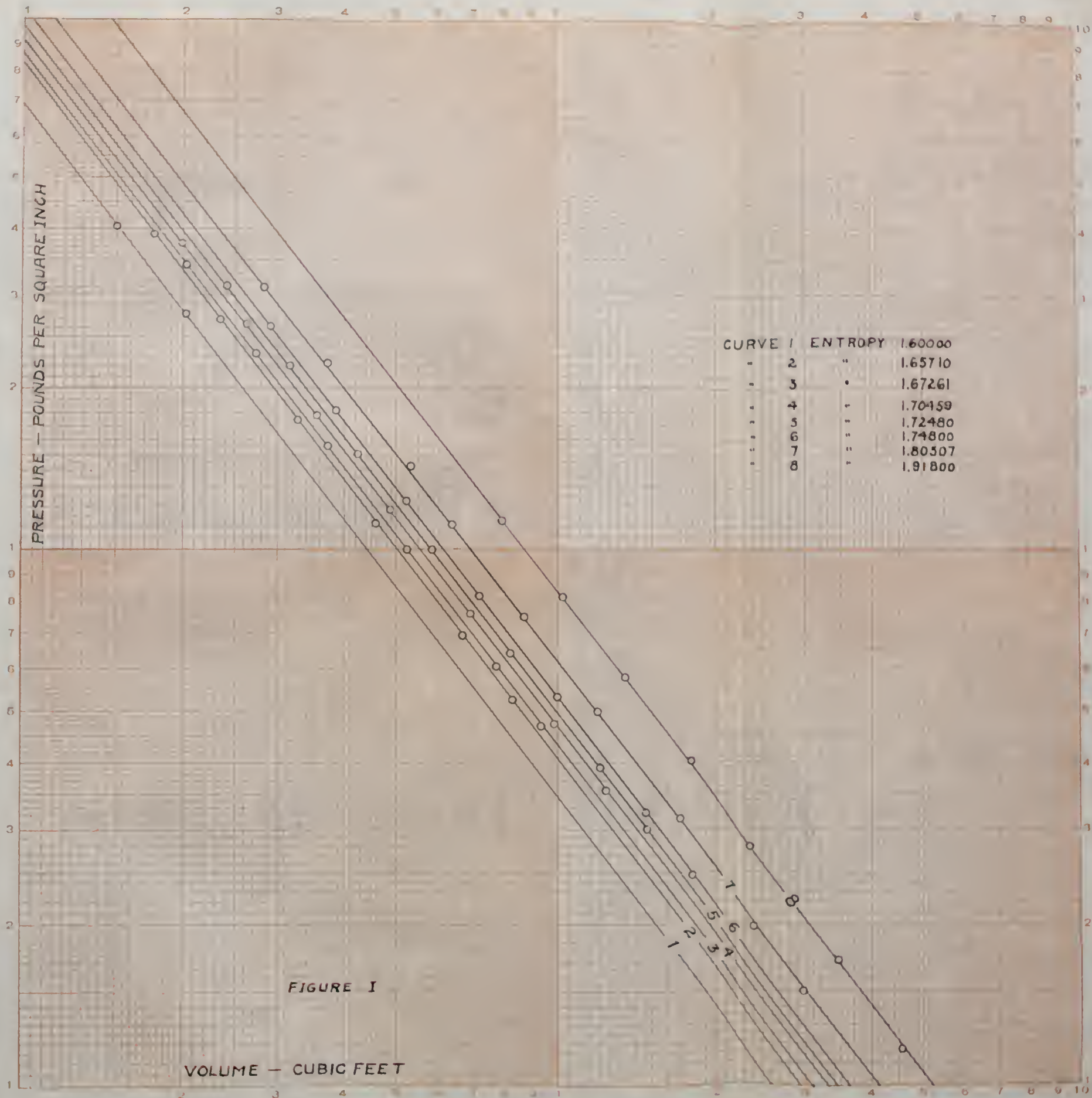
This same process was continued for eight different

values of entropy, varying from 1.60 to 1.918, covering the regions usually employed in practice.

Having pressures and volumes for different entropies, it becomes possible to determine whether or not the relation of p to v is the same throughout the regions under consideration.

To determine the exponent n in the expression $p v^n = C$, which it is intended to establish, pressures and volumes for each entropy used are plotted on logarithmic cross-section paper as shown in Figure 1. The slope of the line drawn through the points corresponding to a particular entropy, gives the desired exponent. The slope is determined by scaling the intercepts of the lines with the axes.





III RESULTS AND CONCLUSIONS.

Eight values of entropy were used in these calculations, and the value of the exponent n corresponding to each is given in table I.

TABLE I

Entropy	Value of n
1.6000	1.301
1.6571	1.303
1.6726	1.300
1.70459	1.293
1.7248	1.294
1.74809	1.298
1.80507	1.283
1.91800	1.290
Mean =	1.295

Then we may write

$$pv^{1.295} = C,$$

also
$$v_2 = v_1 \left[\frac{p_1}{p_2} \right]^{\frac{1}{1.295}}$$

and

$$W = \frac{p_1 v_1 - p_2 v_2}{1.295 - 1} \times 144$$

(if p is in pounds per square inch) for the work in foot pounds which is done during adiabatic expansion from a pressure

p_1 to a pressure p_2 .

As a check upon the accuracy of this exponent, the work in foot pounds is in each case compared with the total change of energy as given by the expression

$$U_1 = T_1 (202.44 + 0.0389T_1) - \frac{cp_1}{T_1^n} (1 + 0.00025 p_1) + 686242$$

$$U_2 = T_2 (202.44 + 0.0389T_2) - \frac{cp_2}{T_2^n} (1 + 0.00025 p_2) + 686242$$

$\log c = 12.20551$ (p in pounds per square inch)

If the exponent just derived for the adiabatic expansion has the proper value, $W = U_1 - U_2$, where U_1 is the total energy in foot pounds at the higher temperature and pressure and U_2 is the total energy in foot pounds at the lower temperature and pressure. The error in per cent of W as compared with $U_1 - U_2$ is given in table II. In table II are given also the upper and the lower pressures and volumes used in making the calculations; also the lower volumes as calculated by using the relation $p_1 v_1^n = p_2 v_2^n$, p_1 , p_2 , and v_1 being known. The error in per cent of this approximate value of v_2 as compared with v_2 calculated by means of equation (3) is also given.

It will be observed from table II that the value 1.3 for the exponent gives results all of which are accurate to within less than one per cent when used at the entropies 1.65 to 1.75, which is the region usually covered in actual practice. At the higher entropies, 1.8 to 1.9, the agreement is not so close when using this value for the exponent. At these higher entropies the value 1.295 gives more accurate results.

This exponent ought in all probability to be some function of the entropy as will be observed from table I. However this would again complicate matters, and thus defeat the purpose of this paper. It seems advisable therefore, to take the value 1.3 as representing nearly enough for all practical purposes the exponent of the adiabatic curve of superheated steam.

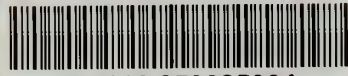
TABLE II.

Entropy	Temperature degrees absolute		Pressures lb. per sq. in.		Volumes - cu. ft.				Work in foot pounds			
	Upper T_1	Lower T_2	Upper p_1	Lower p_2	Upper V_1	Lower V_2		Percent Error	Calculated	Approximate		Error - percent
						Calculated $n = 1.295$	Approximate $n = 1.3$	$n = 1.295$ $n = 1.3$		$n = 1.295$	$n = 1.3$	$n = 1.295$ $n = 1.3$
1.6000	1100	1000	409.50	276.30	1.506	2.035	2.030	-0.706-0.128	2 6708	2 6572	2 6129	-0.51 -2.170
1.6571	1200	800	390.05	69.06	1.764	6.668	6.716	0.720 0.192	108480	111112	109259	2.42 0.715
1.6726	1300	800	490.10	60.83	1.526	7.604	7.644	0.526 0.105	137756	139285	136997	1.11 0.630
1.70459	1300	750	378.70	35.60	1.993	12.265	12.354	0.730 0.180	151809	154609	152030	1.84 0.140
1.7248	1300	750	313.30	30.10	2.422	14.559	14.783	1.540 0.910	152570	156489	153882	2.57 0.860
1.748097	1300	800	261.00	32.18	2.918	14.590	14.663	0.500 0.077	140026	142640	140266	1.86 0.170
1.80507	1500	750	310.00	15.00	2.852	29.502	29.575	0.250-0.720	215400	215486	211963	0.04 -1.59
1.918	1500	750	113.50	55.00	7.845	81.005	80.803	-0.250-0.640	216806	217162	213542	0.16 -1.50





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